Assignment 3

1. Find the linear polynomial interpolating the points (0.3, 4.5) and (0.7, 0.4).

2. Find the quadratic polynomial interpolating the points (0, 5.2), (1, 2.6) and (3, 4.7).

3. Find the quadratic polynomial interpolating the points $(-1, y_{-1})$, $(0, y_0)$ and $(1, y_1)$.

4. What is the condition number of the Vandermonde matrix for finding the interpolating quadratic polynomial interpolating the points $(15235, y_0)$ $(15240, y_1)$ $(15245, y_2)$, and how does that contrast with the condition number of the matrix in Question 3?

5. Up to this point, we have discussed interpolating polynomials. Suppose we had two points (0.3, 4.5) and (0.7, 0.4) but now we wanted to find a trigonometric function of the form $y = a\cos(x) + b\sin(x)$ that passes through these two points. What are the two linear equations that must be solved, and use Matlab to find the coefficients a and b and then test your solution by substituting in $x = 0.3$ and $x = 0.7$ into the resulting trigonometric function.

6. Write down the 1st-order Taylor series for $\sin(2 + h)$. Use this to approximate the value of sin(2.1), and approximate the specific value of ξ so that the formula is exact given that sin(2.1) = 0.8632093666488738 to sixteen significant digits. You can use your calculator and you will need the arc-sine function.

7. Write down the 2nd-order Taylor series for e^{1+h}. Use this to approximate the value of $e^{0.9}$, and approximate the specific value of ξ so that the formula is exact given that $e^{0.9} = 2.459603111156950$ to sixteen significant digits. You can use your calculator and you will need the natural logarithm function.

8. Use some form of mathematical software to plot

$$p(x) = -0.2820860474088350x^2 + 1.162464000058042x - 0.0312849495726095$$

and $\sin(x)$ on the interval [0.4, 0.8]. How would you describe the quadratic polynomial, and how does the error $|p(x) - \sin(x)|$ contrast between [0.5, 0.7] and outside this interval?

9. Suppose you have an array containing 101 entries, you know the entries are sorted, and you understand that they are approximately uniformly distributed. You know that

`array[0] == 5.3` and `array[100] == -8.3`

You are trying to determine if one of the array entries is equal to 0.0. Which entry of the array would you search next if you were using a binary search, and if you were using an interpolating search?

10. Use your calculator to find a point on the interval [10, 15] that equals the average of the values sin(10), sin(11), sin(12), sin(13), sin(14) and sin(15).

11. Does a function have to be just continuous or also differentiable for the intermediate-value theorem to apply?

12. We have the rule that when rounding, if the digit immediately after the digit to which we are rounding is a 5 and all subsequent digits are zero, half the time we round up, and the other half we round down (based on the digit we are rounding to being odd or even, respectively). If we chose to either only round up or only round down in this case, what type of error would this introduce into our calculations?

13. In this course, what are the primary sources of errors we are interested in minimizing?